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AN

APPENDIX

TO

Stereometrical Propositions.

By

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TO
Stereometrical Propositions.

I. Note.

AS an Abstract from the undoubted Axioms of Geometry, it is generally observed, that in a Rank of numbers, having equal difference, the second differences of the squares of those numbers are equal; the third differences of the cubes of those numbers are equal: and so in order in the higher powers. Thus,

In Squares

1

1	1		
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	
1	2	3	4

2

1	1	8	
3	9	16	8
5	25	24	8
7	47	32	8
9	81		
1	2	3	4

Obſerve in the firſt of theſe Examples, in the firſt collum are the numbers of a progreſſition, having equal difference, to wit, a unite. In the ſecond colum, the ſquares of thoſe numbers. In the third colum, the firſt differences. In the fourth colum, the ſecond differences, to wit, 2, 2, 2. In the ſecond Example, in the firſt colum are a Rank of numbers, having equal difference, to wit, 2. In the ſecond colum, their ſquares. In the third colum, the firſt difference. In the fourth colum, the ſecond difference.

II. *Note.*

Hence it follows, that by the help of ſuch differences the table of ſquares may be calculated: thus, in the firſt Example, the ſum of 1 and 3, is 4; the ſquare of 2. The ſum of 2, 3 and 4, is 9; the ſquare of 3. The ſum of 2, 5 and 9, is 16; the ſquare of 4. The ſum of 2, 7 and 16, is 25; the ſquare of 5. The ſum of 2, 11 and 36, is 49; the ſquare of 7. &c.

III.

III. Note.

Like plain numbers are in the same proportion one to another, that a square number is in, to a square number: *Euclide* the 26 Proposition of the Eighth Book. Therefore the second difference in such a Rank of plane numbers are equal. Further, what planes and solids are either equal or proportionable to such Ranks may be gradually calculated; as in the last.

IV. Note.

1	1			
		7		
2	8		12	
		19		6
3	27		18	
		37		6
4	64		24	
		71		
5	125			
1	2	3	4	5

1	1			
		26		
3	27		72	
		98		48
5	125		120	
		218		48
7	343		186	
		386		
9	729			
1	2	3	4	5

In the first Example, in the first colum are the numbers in a Rank having equal difference, to wit, a unite. In the second colum, the cubes of those numbers. In the third colum, the first differences of those cubes. In the fourth colum, the second differences. In the fifth colum, the third differences, to wit, 6, 6. The like in the second Example.

V. Note.

Hence it follows, that the table of cubes may be

made thus : In the first Example, 1 and 7, is 8; the cube of 2. The sum of 8 and 19, is 27; the cube of 3. The sum of 18, 19 and 27, is 64; the cube of 4. The sum of 6, 18, 37 and 64, is 125; the cube of 5. The sum of 6, 24, 61 and 125, is 216; the cube of 6. The sum of 6, 30, 91 and 216, is 343; the cube of 7. The like in the second Example.

VI. Note.

Like solid numbers are in the same proportion one to another, that a cube number is in, to a cube number. *Euclide* the XXVII Prop. of the Eighth Book. Therefore the third differences in such a Rank of solid numbers are equal : further, such planes and solids as are either equal or proportionable to such Ranks, may be gradually calculated, as in the last.

VII. Note.

If a Rank of Squares, whose Roots have equal differences, be multiplied by any number, the second differences of such a Rank of proucts are equal. Let the number multiplying be 10.

0	0	00		
			10	
1	1	10		20
			30	
2	4	40		20
			50	
3	9	90		20
			70	
4	16	160		20
			90	
5	25	250		
1	2	3	4	5

In the first colum are the numbers bearing equal difference. In the second colum are the squares of those numbers. In the third colum the products. In the fourth colum the first differences. In the fifth colum the second differences and they equal.

VIII. Note.

If unto ſuch a Rank of Products, as in the laſt, there be added a Rank of Cubes, whoſe Roots are equal to the Roots of the Squares, the third differences of ſuch a Rank will be equal.

0	00	0	00	11		
1	10	1	11	37	26	
2	40	8	48	69	32	6
3	90	27	117	107	38	6
4	160	64	224	151	44	6
5	250	125	375			
1	2	3	4	5	6	7

In the firſt colum are the numbers having equal difference. In the ſecond colum the Products of their ſquares by a given number. In the third colum the cube of the numbers in the firſt colum. In the fourth colum the ſum of the products and cubes. In the fifth colum their firſt difference. In the fixt colum the ſecond differences. In the ſeventh colum the third differences which are equal.

IX. Note.

Let a conſtant number be added to a Rank of Products, ſo that one of the numbers multiplying

be a conſtant number, and the other of the numbers be the ſquares of numbers having equal difference, and this Rank of ſums be added to a Rank of cubes, whoſe roots are the ſame with the roots of the ſquares; ſuch a compounded Rank will have their third difference equal. Thus,

1	8	10	1	19			
2	8	40	8	56	37		
3	8	90	27	125	69	32	
4	8	160	64	232	107	38	6
5	8	250	125	383	151	44	6
1	2	3	4	5	6	7	8

In the firſt colum are the numbers having equal difference. In the ſecond colum is the conſtant number to be added. In the third colum are the Rank of products, that is, the ſquares of the numbers in the firſt colum multiplied by a given number. In the fourth colum are the cubes of the numbers in the firſt colum. In the fifth colum are the ſum of the numbers in the ſecond, third and fourth colums. In the ſixth colum are the firſt differences of their ſums. In the ſeventh colum are the ſecond differences. In the eighth colum the third differences, and they equal.

X. Note.

In a Rank of numbers, having equal difference, and equal in number; if the third part of the cubes of each of theſe numbers, be ſubſtracted from the products of the ſquares of each of theſe numbers, in half the greateſt number of that Rank, the remainders

mainders will be a Rank of numbers, equal to all the ſquares in the ſeveral portions of one fourth of a ſphere, whoſe diameter is equal to the greateſt number in that Rank, and the third differences of this Rank of portions are equal; but the firſt and ſecond differences will increaſe and decreaſe, differently one from another. Thus,

0	00	00	00			
3	09	108	99	99	162	
6	72	432	360	261	108	54
9	243	972	729	369	54	54
12	756	1728	1152	423	00	54
15	1125	2700	1575	423	54	54
18	1944	3888	1944	369	108	54
21	3087	5292	2205	261	162	54
24	4608	6912	2304	96		
1	2	3	4	5	6	7

In the firſt colum are the numbers having equal difference. In the ſecond colum are the pyramides adſcribed within the cubes of the numbers in the firſt colum. In the third colum are the products of the ſquares of the numbers in the firſt colum, by half the greateſt number in the firſt colum. In the fourth colum are the differences of the numbers in the ſecond and third colums, that is, all the ſquares in ſeveral portions of one fourth of a ſphere, whoſe diameter is 24. In the fifth colum are the firſt differences. In the ſixt colum are their ſecond differences. In the ſeventh colum are the third differences, and they equal.

XI.

XI. Note.

The Application or Use of the Preceeding Notes.

The application or Use may be, to calculate Pyramides and cones, either the whole or their parts; as also to calculate the parabolick and hyperbolick conoides, either the whole, or their frustums; yet also, to calculate the sphere or spheroide, either the whole or their portions or Zones, and that gradually, that is, to find the solidity upon every inch or foot.

To find the solidity of a parabolick Conoide upon every two inches.

To do which, consider the Diagram of the 18 Prop. of my *Stereometrical Prop.* Let P A be 16; A R 12; therefore A V or P H will be 9; for it ought to be as P A, is to A R; so is A R, to A V. Let the axis A P be divided into eight equal parts; *viz.* 2, 4, 6, 8, 10, 12, 14, 16. Let there be planes drawn parallel to the base, through every one of these divisions, though in the Diagram there is not so many. From P to the first Q suppose to be 2, its square 4; the half thereof 2, which multiplied by 9 equal to P H, the Product will be 18; that is, the Prism Q Z G I H P; equal to all the squares in the portion of the conoid Q O P. Let from P, to the second Q be 4, its square 16, the half is 8; which mu ti-

multiplied by 9, the Product is 72; equal to the Prism QZFIHP, equal to all the squares in the portion of the conoid QOP. Let from P to the third Q be 6, its square 36, the half of it is 18, which multiplied by 9, the product is 162; the Prism QZEIHP; equal to all the squares in the third portion of the conoid QOP.

Having obtained two portions, the rest may be obtained thus: having obtained the second difference, which is 36, we may proceed to find the rest by the *Seventh Note*; thus, add 36 to 54, and it makes 90: which added to 72, the sum is 162, equal to all the squares in that portion, and so in order, 36, 90 and 162; the sum is 288. 36, 126 and 288; the sum is 450. 36, 162 and 450, the sum is 648, &c.

0	00		
2	18	18	36
4	72	54	36
6	162	90	36
8	288	126	36
10	450	162	36
12	648	198	36
14	882	234	36
16	1152	270	
1	2	3	4

In the first colum are the parts of the altitude of the conoid. In the second colum are all the squares in several portions of one fourth of a conoid. In the third colum the first differences. In the fourth colum the second differences. These portions in the second colum may be reduced to circular portions, thus, as 14 is to 11; so are all the squares in these portions to the portions themselves.

I. Scholium:

The use of this gradual calculation may be thus: Suppose a Brewers Copper be in form of a parabolick

bolick conoid; the quantity of liquor therein contained may be found, thus, having calculated a table upon every inch, or two inches, or as is thought convenient; then having a ſtraight Ruler divided equally into inches, putting the Ruler into the liquor to the bottom of the Copper, ſee how many inches of the Ruler is wet; with the number of wet inches enter the firſt colum of your table, and in the next colum are the number of cubick inches which that portion contains; the number of cubick inches thus found, being divided by the number of cubick inches in a Gallon, the quotient ſhews the number of Gallons in that portion of the Copper.

II. Scholium:

To compoſe ſeveral works into one.

As 14, is to 11; ſo are all the ſquares in one fourth of the conoid, to one fourth of the conoid it ſelf. becauſe this one fourth ought to be divided by the number of cubick inches in a Gallon, ſuppoſe it 288, to ſhew the number of Gallons in each portion, we may multiply 14 by 288, that is 4032. Then as 4032 is to 11; ſo are all the ſquares in one fourth of the conoid, to the Gallons in that one fourth. Further, becauſe this one fourth ought to be multiplied by 4, to reduce it to a whole conoid; therefore, divide the conſtant diviſor, that is, 4032, by 4, and it will be 1008. Then, as 1008, is to 11; ſo are thoſe ſeveral portions in the ſecond colum of the laſt table, to the num-

number of Gallons in those several portions of a parabolick conoid. By such compositions may the Practitioner compose constant divisors or dividends, which will much breviate the work; this is onely for an Example.

Every parabolick conoid hath its second differences equal. To find the second differences, work thus, Square one of the equal segments of the axis, and multiply that Square by the Parameter, that product will be the second difference. In this Example, the equal segment of the axis is 2; the square of it 4; which multiplied by the Parameter 9, the product is 36, the second difference. Half of the second difference, is always the first of the first difference. Half 36 the second difference, is 18, the first of the first difference, &c. Here note, this 36 is the second difference of one fourth of all the squares in a parabolick conoid; if 36 be multiplied by 4, it makes the second difference 144; whose half is 72, the first of the first differences. Or, the first differences are found by taking half the difference of the squares of any two segments, which multiplied by the Parameter, thereby the first differences are obtained. Thus, to find the first difference answerable to the Segments 6 and 8; the Square of 8, is 64; the Square of 6, is 36; the difference of those Squares is 28, whose half is 14; which multiplied by the Parameter 9, the product is 126; the first difference answerable betwixt 6 and 8.

XII. Note.

To find the solidity of an hyperbolick conoid gradually, to wit, upon every three inches.

For the performance of which, take notice of the XVI Prop. of *Stereom. Prop.* in that Diagram, Let A M equal to A B be 9. Let M L equal to A F, be 6. Let A E be 15 : therefore F E will be 9. Let the rest of the construction be as in that Proposition. Let from M to the first K be 3, whose square is 9, whose half is $4\frac{1}{2}$, the area KHM; which being multiplied by M L, 6; the product will be 27, the prism K H N O L M. Because F E, F C and F L are equal, that is, each of them 9 : Therefore, the first pyramid O N X I L M will be 9. Then this prism and pyramid being added, will make 36, the whole prism K H X I L M, equal to all the squares in the portion K Z M. Let from M to the second K be 6, whose square is 36, its half 18, the area K H M; which being multiplied by M L, 6; the product will be 108, equal to the prism K H N O L M. The cube of 6, is 216; a third part is 72, the pyramid O N X I L, this prism and pyramid being added together, the sum will be 180; the prism K H X I L M : equal to all the squares in the portion K Z M. Let from M to A, be 9; its square 81, the half $40\frac{1}{2}$, equal to the area A B M, which being multiplied by M L, 6 ; the product will be 243 : the cube of 9, is 729; a third part thereof is 243; equal to the pyramid F C D E L : this prism and pyramid being added together, is 486; the whole

whole prism ABDELM, equal to all the squares of the portion AZM.

These three portions being obtained, they may be continued by the VIII *Note*, thus:

0	00			
		36		
3	46		108	
		144		54
6	180		162	
		306		54
9	486		216	
		522		54
12	1008		270	
		792		54
15	1800		324	
		1116		54
18	2216		378	
		1494		54
21	4410		432	
		1926		
24	6336			
1	2	3	4	5

For if the third differences which are equal, and in this Example is 54, be added to the first of the second differences, being 108, it makes 162, and by such additions, the second differences in the fourth colum are made. Further, by adding these second differences to the first of the first differences which is 36, it makes 144, &c. So the numbers in the third colum are made. Yet further, by adding these first differences to the first number in the second colum, the Rank of portions of such a conoid is made.

Then,

By making use of the directions in the first and second *Scholiums*, the number of Gallons are obtained. The parabolick and hyperbolick conoides may well be made use of for Brewers Coppers; the parabolick, when the crown is somewhat blunt; but the hyperbolick conoid when the crown is more sharp.

XIII. Note.

To calculate a sphere gradually, to wit, upon every three Inches.

Consider the XV Prop. of *Stereom. Prop.* Let E D equal to E F, be 24. The rest of the construction as in that Prop. Let from E, to the first R be 3, whose square is 9; whose half is $4\frac{1}{2}$, the area R X E, which being multiplied by 24, the product will be 108; the prism K H X R E F. The cube of 3, is 27; a third part thereof is 9, the pyramid K H O I F; this pyramid taken from the former prism, leaves the prism R X O I F E, 99: equal to all the squares in the portion R Q E. From E, to the second R, 6; its square 36, the half 18, which multiplied by 24, makes 432; the prism R X H K F E. The cube of 6, is 216, a third part of it is 72; the pyramid K H O I F: this pyramid being taken from that prism, there rest 360; the prism R X O I F E, equal to all the squares in the portion RQE. Let from E to the third R be 9; its square 81, the half thereof $40\frac{1}{2}$, the area R X E, this area being multiplied by 24, the product will be 972, the prism KHYREF: the cube of 9, is 729, a third part of it is 243, the pyramid K H O I F: this pyramid being substracted from that prism, the remainder is 729; the prism R X O I F E, equal to all the squares in the third portion R Q E. Having obtained these three portions, the rest may be found by their third difference, according to the X. *Note.*

The

0	00	0	0			
3	09	108	99	99	162	
6	72	432	360	261	108	54
9	243	972	729	369	54	54
12	576	1728	1152	423	00	54
15	1125	2700	1575	423	54	54
18	1944	3888	1944	369	108	54
21	3087	5292	2205	261	162	54
24	4608	6912	2304	99		
1	2	3	4	5	6	7

The numbers in the ſeventh colum are the third differences, and they equal; the numbers in the ſixt colum are the ſecond differences, and are compoſed by ſubſtracting the numbers in the ſeventh from the firſt and laſt numbers in the ſixt colum; the numbers in the fifth colum are the firſt differences, and are compoſed by adding thoſe numbers in the ſixt colum to the firſt and laſt of thoſe in the fifth colum; the numbers in the fourth colum are all the ſquares in ſeveral portions of one fourth of a ſphere whoſe diameter is 24, thoſe portions are made by adding the numbers in the fifth colum to the numbers in the fourth, thus, 261, and 99, is 360. 369, and 360, is 729. 423, and 729, is 1152, &c.

Then making uſe of the firſt and ſecond ſcholium the number of gallons are obtained. Or if it be made, as 54, is to 11, ſo is 54, to a fourth

 number,

number, with that fourth number proceede to make tables of the ſecond and firſt differences, and then the table of portions it ſelfe. Every ſphere hath its third differences equall. To find the third difference, doe thus. Cube one of the equal ſegments of the axis and multiply that cube by 2, and that product will be the third difference, thus, the cube of three is 27, which multiplyed by 2, the product is 54; the third difference of all the ſquares in one fourth of a ſphere. Here note, that it is to be underſtood, that the axis of the ſphere is equally divided into an equal number of ſegments; ſo then, if the number of ſegments in the ſemiaxis, leſs by one; be multiplyed by the third difference, it gives the firſt of the ſecond differences. Thus, the number of ſegments in the ſemiaxis is 4, then 4 leſs 1, is 3; which being multiplyed by 54, the product is 162: the firſt of the ſecond differences.

To find the third difference in one fourth of all the ſquares in a ſpheroid, do thus: The axis being divided as above in the ſphere; cube the difference betwixt two Segments, which being multiplyed by 2, makes a product; then, as the ſquare of the ſemiaxis, is to the ſquare of the other ſemidiameter; ſo is that former product to a fourth number, which will be the third difference. For the ſecond differences, uſe the Rules given for the ſphere.

XIV. *Note.*

To calculate a pyramid or cone gradually. To find the third difference in a pyramid work thus, the

the Altitude of the pyramid being equally divided; cube the difference of the two ſegments, which being doubled, makes a number; then, as the ſquare of the Altitude of the pyramid, is to the area of the baſe of that pyramid; ſo is that former number, to the third difference of that pyramid.

To find the ſecond differences in a pyramid: As the difference of two of the ſegments of the Altitude, is to the following ſegment; ſo is the third difference, to the ſecond difference anſwerable to that ſegment.

To find the firſt differences in a pyramid. Cube two of the ſegments, and take a third part of their difference. Then, as the ſquare of the Altitude of the pyramid; is to the area of the baſe of that pyramid; ſo is that former difference; to the firſt difference anſwerable to thoſe two ſegments.

Let there be a pyramid whoſe Altitude is 10, and one ſide of the baſe is 40, and the other ſide 5; therefore the area of the baſe is 200. Let the Altitude be divided into five equall parts, and to calculate accordingly. To find the third difference, the cube of 2, is 8; whoſe double is 16. Then, as 100 the ſquare of the Altitude, is to 200 the area of the baſe; ſo is 16, to the third difference 32. To find any of the ſecond differences at demand, to find the ſecond difference anſwerable to 8. As 2, the difference betwixt the ſegments 6, and 8, is to 8; ſo is the firſt difference 32, to 128 the ſecond difference anſwerable to 8. The ſecond differences are in proportion one to another, as their anſwering ſegments; as 2, is to

32; so is 8, to 128. To find any of the first differences, cube the two Segments, to wit, 2 and 4, and the cubes will be 8 and 64; then take 8 from 64 and the Remainder is 56, a third part is $18\frac{2}{3}$. then, as the square of the Altitude 100, is to the area of the base 200; so is $18\frac{2}{3}$, to $37\frac{1}{3}$, the first difference answering to 2 and 4. Then by a continuall adding of the third difference to the second differences they are made, and by adding the first of the second differences to the first of the first differences and so in order the first differences are made. Lastly by adding the first differences the Segments of the pyramid, are made according to the III. Note; or thus.

0	0	$5\frac{1}{3}$		
2	$5\frac{1}{3}$	$37\frac{1}{3}$	32	32
4	$42\frac{2}{3}$	$101\frac{1}{3}$	64	32
6	144	$197\frac{1}{3}$	96	32
8	$341\frac{1}{3}$	$325\frac{1}{3}$	128	
10	$666\frac{2}{3}$			
1	2	3	4	5

The numbers in the fifth colum are the third differences, the first number in the fourth colum being found by the Rule before given, all the numbers in that fourth colum may be made by adding the third difference, thus, to 32 adde 32, the summe is 64. adde 32, to 64; the summe is 96. adde 32, to 96; the summe is 128. The first number in the third colum being found by the Rule above, then $5\frac{1}{3}$ added to 32; the summe is $37\frac{1}{3}$.

adde

adde 64, to $37\frac{1}{3}$; the summe is $101\frac{1}{3}$. adde 96, to $101\frac{1}{3}$, the sum is $197\frac{1}{3}$. adde 128, to $197\frac{1}{3}$; the sum is $325\frac{1}{3}$. further, adde the first of the third colum, to the first of the second colum; thus, adde $5\frac{1}{3}$, to 0; the sum will be $5\frac{1}{3}$, adde $57\frac{1}{3}$, to $5\frac{1}{3}$, the sum is $42\frac{2}{3}$, adde $101\frac{1}{3}$. to $42\frac{2}{3}$; the sum is 144. adde $197\frac{1}{3}$, to 144; the sum is $341\frac{1}{3}$, adde $325\frac{1}{3}$, to $341\frac{1}{3}$; the sum is $666\frac{2}{3}$.

If it be to calculate a cone whose diameters of the base are 40 and 5. Let it it be made, as 14, is to 11; so is 32, to the third difference of the same cone. Then proceede with the third difference to make the second and first; and lastly, the table it self.

XV: Note.

The calculation of frustum pyramides whose bases are unlike, To the performance of which consider the third case of the second proposition of *Stereom. Prop.*

Every such solide hath its third differences equall, but the second and first differences will be complicated according to the IX. Note.

To find the third difference proper to the pyramid BCDHF, Let the construction and numbers be the same as in that diagram, and let it be to calculate it upon every two inches, thus. The cube of 2, is 8; the double thereof is 16, Then, as the square of the Altitude 40, that is 1600, is to the area of the base BCDH, 336; so is 16, to $3\frac{36}{100}$. by the Rule delivered in the

14 *Note*, the first of the second differences is $3\frac{36}{100}$. and the first of the first differences is $\frac{56}{100}$.

The solide HDEGVF hath its second differences equall by the VII. *Note*.

To find its first and second differences. The square of 2, is 4. which multiplyed by FV, 26; the product will be 104. then, as 40 the Altitude, is to HD, 28; so is 104, to $72\frac{80}{100}$. the second difference. Therefore the first of the first differences will be $36\frac{40}{100}$.

To find the second differences of the solide ABHOIF the square of 2 is 4, which multiplyed by IF, 30; the product is 120, then, as 40, the Altitude; is to OA, 12: so is 120, to 36, the second difference. Therefore the first of the first differences are 18. For the complication of these differences.

1	2	3	
$\frac{56}{100}$	$3\frac{36}{100}$	$3\frac{36}{100}$	in the pyramid BCDHF
$36\frac{40}{100}$	$72\frac{80}{100}$		in the prism HGEDFV
18	36		in the prism ABHOIF
$54\frac{96}{100}$	$112\frac{16}{100}$	$3\frac{36}{100}$	their summe.

Rejecting the denominators they may be written Thus,

5496	11216	336

Because the denominators are Rejected, therefore the two last figures toward the Right hand are decimals.

0				
2	161496	161496	11216	336
4	334208	172712	11552	336
6	518472	184264	11888	336
8	714624	196152	12224	336
10	923000	208376	12560	336
12	1143936	220936	12896	336
14	1377768	233832	13232	336
16	1624832	247064	13568	336
18	1885464	260632	13904	336
20	2160000	274536	14240	336
22	2448776	288776	14576	336
24	2752128	303352	14912	336
26	3070392	318264	15248	336
28	3403904	333512	15584	336
30	3753000	349096	15920	336
32	4118016	365016	16256	336
34	4499288	381272	16592	336
36	4897152	397864	16928	336
38	5311944	414792	17264	
40	5744000	432056		
1	2	3	4	5

The conſtruction of the table may be thus; the numbers in the firſt colum are the third differences, The firſt number in the fourth colum is the complicated ſecond difference, and the other number in that fourth colum are made thus, to the firſt 11216, adde 336; the ſum is 11552. Then to that 11552, adde 336; the ſum is 11888, &c.

The firſt number in the third colum is complicated from the firſt complicated difference and a parallelipepidon whoſe baſe is the plane RIFV, and the Altitude the firſt Segment of the Altitude of the fruſtum, thus, the plane RIFV, is 780; which being doubled is 1560; then, 156000 more 5496 is 161496; the firſt of the firſt differences, then 161496 more 11216, is 172712. Further, 172712 more 11552, is 184264. Yet farther 184264 more 11888, is 196152, &c.

The numbers in the ſecond colum are made thus, the firſt number in the ſecond colum, is the ſame as the firſt number in the third colum, then, 161496 more 172712, is 334208, and 334208 more 184264, is 518472, &c.

Then makeing uſe of the firſt and ſecond ſcholium, the quantity of Liquor that ſuch veſſels contain may eaſily be obtained.

XVI. *Note.*

To calculate Elliptick ſolides whoſe baſes are unlike. The calculation of ſuch ſolides are the ſame as in the 15, note for if the firſt, ſecond and third complicated differences be found, then makeing uſe of this propotion as 14, is to 11; ſo is 336; to the third difference. And

And

As 14, is to 11; so is 11216, to the first of the second differences.

Further

As 14, is to 11; so is 161496, to the first of the first differences, then proceede to make the table it self, as in the 15 note. Or make use of the secund scholium of the 11 note and you will have the quantity in Gallons.

Or

Such Elliptick solides may be calculated by the 12 note: for every such Elliptick solide is equall to a frustum hyperbolick conoide whose circular bases of the conoide, are equall to the Elliptick bases of the Elliptick solide; and the Altitude of one frustum is equall to the Altitude of the other.

XVII. *Note.*

Every hyperbolick conoid hath its third differences equal.

To find the third, second and first differences in an hyperbolick conoid, and consequently to calculate that conoid gradually. In the forementioned diagram of the 17. prop. Stereom. Prop. Let GM, the Transverse diameter be 12. ML, the parameter 6. MA, the axis of the conoid 24.

To

To calculate the ſolidity of this conoid vpon every three Inches.

To find the third difference of this conoid.

Take the difference of two of the Segments, to wit, 3; whoſe cube is 27: whoſe double is 54. Then, as GM, 12; is to ML, 6: ſo is 54, to 27. The third difference of all the ſquares in one fourth of that conoid proper to that pyramid FCDEL.

By the Rule in the laſt note the firſt of the ſecond differences is 27.

For the firſt of the firſt differences, worke thus; take the firſt Segment which is 3; whoſe cube is 27: a third part is 9, then, as GM, 12; is to ML, 6: ſo is 9, to $4\frac{1}{2}$. the firſt of the firſt differences proper to the pyramid FCDEL.

The ſecond and firſt differences of all the ſquares in one fourth of this conoid, is complicated from the ſecond and firſt differences of the pyramid FCDEL, and the ſecond and firſt differences of the priſm ABCFLM. Every ſuch priſm hath it ſecond difference equall.

To find the ſecond and firſt difference of the priſm ABCFLM.

Square the difference of two of the Segments of the axis, to wit, 3; that is 9, which being multiplyed by the parameter ML, 6; the product is 54, the ſecond difference. The firſt of the firſt differences of every ſuch priſm is half of the ſecond

second difference; therefore the first of the first differences is 27.

To complicate these differences.

1	2	3	differences
$4\frac{1}{2}$	27	27	in the prism FCDEL.
27	54		in the prism ABCFLM.
$31\frac{1}{2}$	81	27	in all the squares of one fourth of that hyperbolick conoid.

0	0			
		$31\frac{1}{2}$		
3	$31\frac{1}{2}$		81	
		$112\frac{1}{2}$		27
6	144		108	
		$220\frac{1}{2}$		27
9	$364\frac{1}{2}$		135	
		$355\frac{1}{2}$		27
12	720		162	
		$517\frac{1}{2}$		27
15	$1237\frac{1}{2}$		189	
		$706\frac{1}{2}$		27
18	1944		216	
		$922\frac{1}{2}$		27
21	$2866\frac{1}{2}$		243	
		$1165\frac{1}{2}$		
24	4032			
1	2	3	4	5

In the first colum are the third differences. In the fourth colum the second differences. In the third colum the first differences, In the second colum the portions of all the squares of one fourth of an hyperbolick conoid, upon every three inches,

inches, whose Transverse diameter is 12, and parameter is 6, and axis is 24.

The construction of this table is the same as the former; thus, 81 more 27; is 108. more 27; is 135. more 27, is 162. &c.

$31\frac{1}{2}$. more 81; is $112\frac{1}{2}$. more 108; is $220\frac{1}{2}$. &c.

0 more $31\frac{1}{2}$; is $31\frac{1}{2}$. more $112\frac{1}{2}$; is 144. more $220\frac{1}{2}$. is $364\frac{1}{2}$. more $355\frac{1}{2}$; is 720.

Here remember that the Transverse diameter is found, by the 9 of the 23 Proposition. of *Stereom. Prop.* Also the parameter found by the converse of the first part of the 11 Prop. of *Stereom. Prop.* The parameter of the parabolike conoid is found, by the converse of the 9 Prop. Of *Stereom. Prop.*

XVIII. Note.

Cautions Concerning Reduction.

I

If it be to calculate pyramids whether Regular or Irregular, whole or frustums; the third, second and first differences are to be found as above: then Reduce those differences into Gallons and parts of a Gallon, or Barrells, or parts of a barrels;

Thus

Suppose 288 cubick inches make one Gallon, and 36 Gallons make one Barrell.

Then,

If the measure be taken in inches, divide the third, second and first differences by 288, and so there will be three quotients in Gallons or parts of

of a Gallon; then with those three quotients proceede to make the table of solid Segments, and that table will be in Gallons or parts of a Gallon. If it be to calculate a table in Barrells multiply 288 by 36 and the product will be 10368 the number of cubick inches in one Barrell. Then divide the third, second and first differences by 10368, there will be three quotients in Barrells or parts of a Barrell: Then with these three quotients proceede to make the table of solides Segments.

That table being so made will be in Barrells or parts of a Barrell.

2,

To calculate Cones and Elliptick solids, whether the whole or their frustums.

Haveing found their third second and first differences, as above, and it be to calculate them in cubick inches, Let it be made as 14, is to 11; so is the third difference, to a fourth,

And,

As 14, is to 11; so is the second difference, to a fourth,

Further,

As 14, is to 11; so is the first difference, to a fourth with these three number thus found, proceede to make the table of solid Segments, and that table will be in cubick inches.

To calculate these solids in Gallons.

Multiply 14 by 288 the product will be 4032.

Then,

As 4032, to 11; so is the third difference, to a fourth.

And

And,

As 4032, to 11; so is the second difference, to a fourth.

Further,

As 4032, to 11; so is the first difference, to a fourth.

With these three numbers thus found, proceed to make the table of solid Segments. So that table will be in Gallons.

To calculate these solids in Barrells.

Suppose 288 cubick inches makes one Gallon, and 36 gallons makes one Barrell, then multiply 288, 36 and 14 one into another and they make 145152.

Then,

As 145152, is to 11; so is the third difference, to a fourth.

And,

As 145152, is to 11; so is the second difference, to a fourth.

Further,

As 145152, is to 11; so is the first difference, to a foruth.

With these three numbers thus found make the table of solid Segments: that table will be in Barrells.

III.

Having found the third, second and first differences of all the squares of one fourth of a sphere, spheroid and hyperbolick Conoid, as in the 12 and 13 notes and the second and first differences of all the squares of one fourth of a parabolick conoid as in the 11 note: they may be Reduced to Circular differences.

Thus,

Thus,

As 14, is to 11; ſo is the third difference, to a fourth.

And,

As 14, is to 11, ſo is the ſecond difference, to a fourth.

Further,

As 14, is to 11; ſo is the firſt difference, to a fourth.

With theſe numbers thus found make a table, of ſolid Segments of cubical inches of one fourth of any of theſe ſolids.

Theſe ſolid Segments ought to be multiplyed by four, to reduce them to ſolid Segments of a whole ſphere, ſpheroid, hyperbolick and parabolick conoid: but to ſhun that work divide 14, by four, and then find the new differences; but becauſe 14 cannot be juſt divided by four, therefore divide 14, by two, and multiply 11, by two, and then work; Thus,

Then,

As 7, to 22; ſo is that third difference, to a fourth.

And,

As 7 to 22; ſo is that ſecond difference, to a fourth.

Further,

As 7, to 22; ſo is that firſt difference, to a fourth.

With theſe numbers thus found, proceed to make tables as is taught in thoſe Notes: tables ſo made, will be tables of ſolid Segments of thoſe ſolids, in cubick inches.

To calculate theſe ſolids in Gallons.

Multiply

Multiply 288 by 14, whose product is 4032; one fourth thereof is 1008;

Then,

As 1008, is to 11; so is the third difference, to a fourth.

And

As 1008, is to 11; so is the second difference, to a fourth.

Further,

As 1008, is to 11; so is the first difference, to a fourth.

Tables being made, with numbers thus found; according to the former directions in the sphere, spheroid, hyperbolick and parabolick conoids, will be tables of solid Segments of a whole sphere, spheroid, hyperbolick and parabolick conoid, in Gallons or parts thereof.

To calculate these solids in Barrells.

Multiply 4032 by 36, the product will be 145152, one fourth thereof will be 36288;

Then,

As 36288, is to 11; so is the third difference, to a fourth.

And,

As 36288, is to 11; so is the second difference, to a fourth.

Further,

As 36288, is to 11; so is the first difference, to a fourth.

Tables being made, with numbers thus found, according to the former directions, will be tables of solid Segments in Barrells. &c.

Then,

Then,

Using a Rod or Ruler equally divided into inches as in scholium the first, the number of Gallons or Barrells may speedily be obtained.

As for the just magnitude of the Gallon, it is not my businesse to dispute; that being determined by custom or Authority: I took 288 onely for Example sake.

XIX. Note.

In a Rank of numbers having equal differences.

Let the first term in the Rank be Z, its square ZZ. the second term 2Z, its square 4ZZ, therefore the first of the first differences is 3ZZ, the third term in that Rank 3Z, its square 9ZZ, then 9ZZ, Less 4ZZ, the second of the first differences 5ZZ, therefore 5ZZ Less 3ZZ the second difference will be 2ZZ.

Further,

The fourth term in that Rank is 4Z, its square is 16ZZ, then 16ZZ Less 9ZZ the third of the first differences 7ZZ; again, 7ZZ Less 5ZZ the second difference is 2ZZ.

Hence it follows,

That the second difference is equall to the square of the first term doubled.

Or also,

The second difference is equall to the square

of

of the difference of two of the terms, (in order taken) doubled.

By the ſame method we find that the third differences in a Rank of cubes are equall, and the third difference is equal to the firſt term multiplyed by 6.

Or,

The third difference is equal to the cube of the difference of two of the terms, taken in order, multiplyed by 6.

The index and equal difference, of every power agrees; to wit, the index of the ſquare is 2, and the ſecond differences are equal. The index of the cube is 3, and the third differences are equal. The index of the ſquare ſquared is 4, and the fourth differences are equal. &c.

The equal difference of every power, is complicated from the index of that power, and the equal difference of the next Leſſer power.

Let the Rank be in naturall order, Thus; 1, 2, 3, 4. &c.

The indices of the powers, Thus.

1	2	3	4	5
Z	ZZ	ZZZ	ZZZZ	ZZZZZ

A unity the equal difference in that naturall Rank, whoſe ſquare is 1, which multiplyed by 2 the index of the ſquare the product is 2, the equal difference in the ſquares: 3, the index of the cube multiplyed by 2 the equal difference in the ſquares, the

the product is 6, the equal difference in the cubes. 4 the index of the square squared multiplyed by 6 the product is 24 the equall difference in the square squared, &c.

If the Rank be in order thus, 2, 4, 6, 8, &c.

2 the equal difference of this Rank whose square is 4; multiplyed by 2 the index of the square the product is 8; the equal difference of the squares in such a Rank. Because the equal difference of the Rank is 2, therefore the indices are to be doubled, &c.

And the equall differences of the powers in such a Rank will be 8, 48, 384, &c.

XX. Note.

For the more easier calculation of the second sections of the sphere and spheroid; worke, Thus.

From the double of the superficies of the triangle BZN, substract the superficies of the triangles BZGD and NZPA, the Remainder will be the superficies of the triangles BZPA and NZGD, the areas of these two triangles being substracted from the area of the triangle NZB, the Remainder will be the superfice of the triangle ZGDAP.

FINIS.

Faults Escaped in the Impression of *Stereometrical Propositions.*

Page 1, line 18, for and Z Read and H. p. 10, l. 23, for 56, r. 58. p. 34, l. 25, after RI put. p. 43, for 297232, r. 297432. p. 45, for 18, r. √432. p. 46, l. 1, for XVI, r. XVII. and l. 21, for AE, 16; r. AF, 6; p. 48, l. 6, for 634. r. 624. p. 51. l. 22, for diameter, r. semidiameter. p. 58, l. 25, for ZB, r. XB; p. 63, l. 1, for XIII. r. XXIII. p. 100, l. 17, for parameter, r. diameter. p. 102 and 103 for as 4 to 3, r. as 3 to 2. p. 105, l. vlt. for Z+2. r. Z−2. p. 106, l. 4, for Z=$\frac{1}{1}$, r. Z−$\frac{1}{3}$. and l. 25, for 89, r. 98.

p. 7 against 12 in the first col. in the sec. r. 576. and in the fifth colum f. 96 r. 99. p. 13. in the second col. f. 46. r. 36. and in the same col. f. 2216, r. 2916.

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